IMPORTANT NOTICE:

This packet contains a review of material that you are expected to have mastered. It is strongly recommended that you read this packet in its entirety and complete all practice problems. We will review this packet on the first day of class.

A TEST ON THIS MATERIAL WILL BE GIVEN THE FIRST MATH TEST DAY.

TOPICS:

(1) Finding the Equation of a Line
(2) Solving Systems of Linear Equations
(3) Factoring Quadratic Expressions
(4) Solving Quadratic Equations

The equation of a line can be written in three forms:

**Standard Form:**

\[ Ax + By = C \]

where (1) A, B and C are integers; and (2) A and B aren’t both zero.

**Slope-Intercept Form:**

\[ y = mx + b \]

where (0, b) is the y-intercept.

**Point-Slope Form:**

\[ (y - y_1) = m(x - x_1) \]

where \((x_1, y_1)\) is any point on the line.

Sophomores should be familiar with all 3 forms. Some freshmen may not be familiar with point-slope form but all should be familiar with the other 2 forms. Despite exposure to more than one form, most students are most comfortable with Slope-Intercept Form. In fact, many students are not exposed to the other forms until high school. I agree that Slope-Intercept form is the most useful for graphing lines on an x-y coordinate plane. When writing the equation of a line, however, Point-Slope Form is more efficient.
Here is how it is done:

**EXAMPLE:** Write the equation of the line through $A(-5, 3)$ and $B(7,1)$.

**POINT-SLOPE FORM**

**Step 1:** Find slope using $m = \frac{y_2 - y_1}{x_2 - x_1}$.

Here, $m = \frac{1 - 3}{7 - (-5)} = \frac{-2}{12} = \frac{-1}{6}$.

**NOTES:**
- It doesn’t matter which point is Point 1 or Point 2.
- Watch for the double negative.
- Reduce the fraction.
- Denominators of 1 should not be written.

**Step 2:** Plug in slope and ONE point into Point-Slope Form.

Using $m = \frac{-1}{6}$ and $B(7, 1)$, we get $(y-1) = \frac{-1}{6}(x-7)$

**NOTES:**
- Only one point should be used.
- The equation needs an “$x$” and a “$y$”

Unless instructed otherwise, this is an acceptable form for the equation of a line. Re-read the PROBLEM set forth at the top of this page. It does not specify a specific form. Use point-slope form because it is the most easily and most quickly attained. In addition, distributing and adding fractions may result in arithmetic mistakes—mistakes that will not be made if the equation is left in point-slope form.

**WARNING:** Many students prefer to rely solely on slope-intercept form. These students plug in the slope and a point and “solve for $b$,” the $y$-coordinate of the $y$-intercept. This process is inefficient and will not be permitted in Honors Geometry.
SLOPE-INTERCEPT FORM

If asked to write the equation in Slope-Intercept Form, perform the following steps:

Step 3: Apply the distributive property to distribute the slope.

\[(y-1) = \frac{-1}{6} \cdot x + \frac{7}{6}\]

**NOTE:** Watch for the double negative.

Step 4: Isolate \(y\) using the addition property.

\[y = \frac{-1}{6} \cdot x + \frac{7}{6} + 1\]

Step 5: Combine like terms by finding a common denominator.

\[y = \frac{-1}{6} \cdot x + \frac{13}{6}\]

**NOTE:** The \(y\)-intercept is \(0, \frac{13}{6}\).

STANDARD FORM

If asked to write the equation in Standard Form, perform the following steps:

Step 6: Clear the denominator by multiplying **both** sides of the equation by the Least Common Multiple of all of the denominators.

\[6y = \frac{-1}{1} \cdot x + 13\]

**NOTE:** This is necessary because \(A, B\) and \(C\) must be **integers**!

Step 7: Use the addition/subtraction property to bring the \(x\)-term to the left side.

\[1x + 6y = 13\]

**NOTES:**
- The \(x\)-term **must** come before the \(y\)-term.
- The coefficient of \(1\) need **not** be written.
PRACTICE:
Write the equations in all 3 forms for the lines going through the pairs of points below.
1.1 \( C(5, 9); \ D(3, -2) \)
1.2 \( E(-11, 4); \ F(-5, -1) \)
1.3 \( G(24, -20); \ H(6, 4) \)
1.4 \( J(0, 3); \ K(1, -1) \)

Reminder:
Although starting with point-slope form may not always be necessary, using slope-intercept form to “solve for \( b \)” is not acceptable.
SPECIAL CASES

Even students who exhibit exceptional algebra skills tend to have difficulties when it comes to identifying and writing equations for horizontal and vertical lines. We will review these topics during the first few days. Nevertheless, here is a quick review.

HORIZONTAL LINES

Horizontal lines have $m = 0$. Therefore, when $m$ is plugged into Point-Slope Form, the x-term goes "away"—that is, it goes to zero.

$$ (y - y_1) = 0(x - x_1) $$

$$ (y - y_1) = 0 $$

$$ y = y_1 $$

Some students have difficulty identifying horizontal lines because they mechanically apply the steps set forth above. Here is a good rule-of-thumb to save time:

**HORIZONTAL LINES**

If both points have the same y-coordinate, the line is horizontal and the equation of that line is \( y = y_1 \).

**Example:** Find the equation of the line through \( P \left(3, \bar{8}\right) \) and \( R \left(117, \bar{8}\right) \).

While students can find slope and use point-slope form, recognizing the common y-coordinate makes the process much faster. Here the equation of the line is \( y = \bar{8} \).

If a student fails to see the common y-coordinate, he will likely waste time going through the following:

$$ m = \frac{\bar{8} - \bar{8}}{117 - 3} = \frac{0}{114} $$

$$ (y - \bar{8}) = \frac{0}{114}(x - 3) $$

$$ y + 8 = \frac{0}{114}x - 0 $$

$$ y + 8 = 0 $$

$$ y = \bar{8} $$
VERTICAL LINES

The term “slope” is undefined for vertical lines. It is best to state that vertical lines have “no slope.” If a line has no slope, students may not use point-slope form. There is NO slope. Also, students may not use slope-intercept form. There is NO slope. But there is an equation. So, what should you do?

First, students must learn to recognize a vertical line. Here is a good rule-of-thumb:

VERTICAL LINES

If both points have the same x-coordinate, the line is **vertical** and the equation of that line is \( x = x_1 \).

**Example:** Find the equation of the line through \( V(6, -256) \) and \( W(6, 103) \).

Common x-coordinate \( \rightarrow \) No Slope \( \rightarrow \) No Point-Slope \( \rightarrow \) No Slope-Intercept \( \rightarrow \) No \( “y=mx+b” \)

**IF NO Y=**, then use \( X= \). Here, the equation of \( x = 6 \).

**PRACTICE:**
Write the equations for the lines going through the pairs of points below.

1.5 \( S(2,11); T(-6,11) \)
1.6 \( A(-13,7); B(-13, -8) \)
1.7 \( J(6, -27); K(6, 14) \)
1.8 \( L(0, 17); M(0, 2) \)
TOPIC: Solving Systems of Linear Equations

Students should be familiar with two methods for solving systems of linear equations: (1) Substitution and (2) Elimination.

**SUBSTITUTION**

Substitution requires a student to isolate **ONE** variable in **ONE** equation. The expression on the other side of the equal sign is then plugged into the **OTHER** equation in order to solve for the **OTHER** variable.

**Example:**

\[
\begin{align*}
3x - 2y &= 19; \\
2x + y &= 8
\end{align*}
\]

**Step 1:** Choose which variable in which equation to isolate. Here, it is best to choose to isolate \( y \) in the 2\(^{nd} \) equation. Why? Applying the subtraction property, the 2\(^{nd} \) equation may be written as:

\[ y = -2x + 8. \]

**Step 2:** Plug-in the new expression for \( y \) in the 1\(^{st} \) equation.

\[ 3x - 2(-2x + 8) = 19 \]

**NOTE:** Plug-in with parentheses!

**Step 3:** Distribute.

\[ 3x + 4x - 16 = 19 \]

**Step 4:** Combine like terms.

\[ 7x - 16 = 19 \]

**Step 5:** Apply algebraic properties (addition, subtraction, multiplication & division) to solve for \( x \).

\[ x = 5 \]

**Step 6:** Plug \( x = 5 \) into either equation or, more efficiently, into the expression found in Step 1 to solve for \( y \).

\[ y = -2(5) + 8 \]

\[ y = -10 + 8 \]

\[ y = -2 \]
ELIMINATION

Elimination, also called Linear Combination, requires a student either to add or to subtract the equations of the system in order to eliminate ONE variable.

Example: \[
\begin{align*}
3x - 2y &= 19; \\
2x + y &= 8
\end{align*}
\]

Step 1: Choose a variable to eliminate. Here, choose to eliminate y.

Step 2: Multiply one or both equations in order to attain common coefficients for the variable to be eliminated.

\[
\begin{align*}
3x - 2y &= 19; \\
2 \cdot (2x + y &= 8) \implies 3x - 2y &= 19; \\
4x + 2y &= 16
\end{align*}
\]

Step 3: Decide how to combine these equations and do so. Addition? Subtraction?

\[
\begin{align*}
3x - 2y &= 19; \\
+ [4x + 2y &= 16]
\end{align*}
\]

\[
7x = 35
\]

Step 4: Apply the multiplication or division property to solve for the OTHER variable.

\[
x = 5
\]

Step 5: Plug \(x = 5\) into either equation to solve for y.

\[
\begin{align*}
2(5) + y &= 8 \\
10 + y &= 8 \\
y &= -2
\end{align*}
\]
PRACTICE:
Solve each system. Vary the method according to which is more efficient.

2.1 \[
\begin{align*}
\begin{cases}
x + y &= -2 \\
3x + y &= 4
\end{cases}
\end{align*}
\]

2.2 \[
\begin{align*}
\begin{cases}
4x + 3y &= 38 \\
2x + y &= 16
\end{cases}
\end{align*}
\]

2.3 \[
\begin{align*}
\begin{cases}
y &= -5x \\
2x - 3y &= 17
\end{cases}
\end{align*}
\]

2.4 \[
\begin{align*}
\begin{cases}
3x + 2y &= 0 \\
8x + 7y &= 5
\end{cases}
\end{align*}
\]

2.5 \[
\begin{align*}
\begin{cases}
x - 6y &= -10 \\
3x + 2y &= 10
\end{cases}
\end{align*}
\]

2.6 \[
\begin{align*}
\begin{cases}
y &= x + 3 \\
4x - y &= 12
\end{cases}
\end{align*}
\]

2.7 \[
\begin{align*}
\begin{cases}
x - 2y &= 5 \\
3x + 4y &= 35
\end{cases}
\end{align*}
\]

2.8 \[
\begin{align*}
\begin{cases}
2x - 11y &= 5 \\
2y - 5x &= 13
\end{cases}
\end{align*}
\]

2.9 \[
\begin{align*}
\begin{cases}
3x + 2y &= 12 \\
7y &= x - 4
\end{cases}
\end{align*}
\]
TOPIC: Factoring Quadratic Expressions

PRACTICE:
Factor each quadratic expression.

3.1 \(x^2 + 14x + 13\)
3.2 \(x^2 - 14x + 24\)
3.3 \(x^2 - 15x + 36\)
3.4 \(x^2 + 7x - 30\)
3.5 \(x^2 + 3x - 18\)
3.6 \(x^2 + 8x + 16\)
3.7 \(x^2 - 49\)
3.8 \(2x^2 - 5x - 12\)
3.9 \(z^2 + 7z + 10\)
3.10 \(9x^2 - 30x + 25\)
3.11 \(2x^2 - x - 21\)
3.12 \(x^2 + 22x + 72\)
3.13 \(3x^2 - 12\)
3.14 \(x^2 + 10x + 24\)
3.15 \(11x^2 - 54x - 5\)
3.16 \(6x^2 - 19x + 15\)
3.17 \(4x^2 - 12x\)
3.18 \(x^2 - 6x - 27\)
3.19 \(x^2 + 3x + 2\)
3.20 \(7x^2 - 14x + 7\)
TOPIC: Solving Quadratic Equations

Students should be familiar with two methods\(^1\) for solving quadratic equations: (1) Solving by Factoring and (2) Solving by the Quadratic Formula. Both methods require a student to use algebraic properties to manipulate the equation into standard form for a quadratic equation.

STANDARD FORM OF A QUADRATIC EQUATION

\[ ax^2 + bx + c = 0 \]

where \(a \neq 0\).

NOTES:

- Many students ignore the condition that \(a \neq 0\). This is necessary because were \(a = 0\), the equation would not be a quadratic. It would be linear.

- The values of \(a\), \(b\) and \(c\) may be negative or positive. The “+” signs used in writing standard form, however, do not change.

- This should not be confused with Standard Form for the Equation of a Line. The “\(b\)” used in the Standard Form of a Quadratic Equation is not the \(y\)-coordinate of the \(y\)-intercept.

\(^1\) A third method, Completing the Square, will be introduced during the course but will not be covered herein.
SOLVING BY FACTORING

EXAMPLE: Solve $5x^2 - 7x - 9 = -11$.

Step 1: Place equation in standard form.

$$5x^2 - 7x + 2 = 0$$

Step 2: Factor.

$$(5x - 2)(x - 1) = 0$$

Step 3: Apply the Zero Product Rule.

*The Zero Product Rule*

If two (or more) numbers are multiplied together such that their product is zero, then at least one of the numbers is zero.

So, here if $(5x - 2)(x - 1) = 0$, then either $(5x - 2) = 0$ or $(x - 1) = 0$.

Step 4: Solve both linear equations.

$$(5x - 2) = 0$$
$$5x = 2$$
$$x = \frac{2}{5}$$

OR

$$(x - 1) = 0$$
$$x = 1$$

The solution should be written as $x = \left\{ \frac{2}{5}, 1 \right\}$.  

*NOTE:* Use braces; do not use parentheses.
PRACTICE:
Solve by factoring,

4.1 \(3x^2 + 7x + 4 = 10\)
4.2 \(x^2 + 5x = 6x + 12\)
4.3 \(x^2 + 15x + 36 = 0\)
4.4 \(x^2 = 100\)
4.5 \(x^2 - 16 = 6x\)
SOLVING BY THE QUADRATIC FORMULA

The Quadratic Formula

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

EXAMPLE: \( 3x^2 + 5x + 3 = 5 \)

Step 1: Place equation in standard form.

\( 3x^2 + 5x - 2 = 0 \)

Step 2: Identify a, b and c.

\( a = 3; \quad b = 5; \quad c = -2 \)

NOTE: Be sure to note NEGATIVE values.

Step 3: Write the quadratic formula substituting these values for a, b and c.

\[ x = \frac{-(5) \pm \sqrt{(5)^2 - 4(3)(-2)}}{2(3)} \]

NOTE: Parentheses are recommended.
Step 4: Simplify and reduce.

\[ x = \frac{-(5) \pm \sqrt{(5)^2 - 4(3)(-2)}}{2(3)} = \frac{-5 \pm \sqrt{25 + 24}}{6} \]

\[ = \frac{-5 \pm \sqrt{49}}{6} \]

\[ = \frac{-5 \pm 7}{6} \]

\[ = \frac{-12}{6} \text{ OR } \frac{2}{6} \]

Present the solution as \( x = \left\{ \frac{-2}{3} \right\} \)

NOTES:
(1) Catch the double negative.
(2) Reduce fractions and radicals.
(3) Braces.
PRACTICE:
Solve using the quadratic formula. Keep all answers in reduced radical form.

4.6 $2x^2 + 7x + 2 = 0$
4.7 $-x^2 - 9x + 5 = 0$
4.8 $x^2 = 3x + 4$
4.9 $4x^2 + 7x - 5 = 4$
4.10 $2x^2 + 11x + 10 = 0$